

classical mechanics goldstein solutions chapter 3

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Introduction

Goldstein's *Classical Mechanics* is a staple reference for graduate students and advanced undergraduates tackling the intricacies of dynamics. Among its many chapters, Chapter 3—dedicated to the Lagrangian formalism—serves as a pivotal bridge between Newtonian mechanics and modern analytical techniques. For those seeking to deepen their understanding, a comprehensive set of solutions to the problems in this chapter is invaluable. This article presents a thorough exploration of the key concepts covered in Chapter 3, addresses common challenges students face, and offers detailed, step by step solutions to selected problems. By following this guide, readers will gain confidence in applying the Lagrangian approach to a wide variety of physical systems.

Goldstein's Classical Mechanics: A Primer

First published in 1950, Goldstein's text has evolved through multiple editions, each refining the exposition of classical mechanics. The book is structured around three foundational frameworks: Newtonian, Lagrangian, and Hamiltonian mechanics. While the Newtonian section introduces the familiar force–mass–acceleration relationship, the Lagrangian section—Chapter 3—offers a more elegant, generalized method that excels in complex coordinate systems and constrained motions.

Students often find the transition from Newton's second law to the Euler–Lagrange equation challenging. However, once the principle of least action is internalized, the Lagrangian formalism reveals a powerful, unifying language that can describe systems ranging from a simple pendulum to a rotating rigid body.

Chapter 3 Overview: The Lagrangian Formalism

Chapter 3 is organized around the following themes:

- Introduction to generalized coordinates and the configuration space.
- Derivation of the Euler–Lagrange equations from the principle of stationary action.
- Discussion of cyclic coordinates and conserved quantities.
- Application of the formalism to simple mechanical systems.

Each section builds upon the last, culminating in a set of practice problems that test a student's ability to translate physical intuition into mathematical expressions.

Key Concepts Covered in Chapter 3

Generalized Coordinates

In classical mechanics, the state of a system with N degrees of freedom can be described by a set of coordinates $q_1, q_2, \dots, q_N$. Unlike Cartesian coordinates, generalized coordinates can be any parameters that uniquely define the system's configuration, such as angles, arc lengths, or even time-dependent functions. This flexibility allows for the efficient description of constrained systems.

For example, a simple pendulum can be described by the angle θ between the rod and the vertical, rather than the Cartesian coordinates of the bob. This choice simplifies the kinetic and potential energy expressions and leads

directly to the Euler–Lagrange equations.

Euler–Lagrange Equations

The cornerstone of the Lagrangian approach is the Euler–Lagrange equation:

Equation: $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0$

where $L = T - V$ is the Lagrangian, the difference between kinetic energy T and potential energy V . By applying this equation to each generalized coordinate, one obtains a set of coupled differential equations that fully describe the system's dynamics.

Conservation Laws

When a coordinate does not appear explicitly in the Lagrangian—i.e., it is cyclic—the corresponding generalized momentum is conserved. This principle is a direct consequence of Noether's theorem, linking symmetries to conservation laws. In Chapter 3, students learn to identify cyclic coordinates and use the resulting constants of motion to simplify problems.

Examples and Applications

Goldstein's text intersperses theory with practical examples, such as:

- Linear motion under constant force.
- Harmonic oscillators in one and two dimensions.
- Central force problems leading to planetary orbits.
- Rotational dynamics of rigid bodies.

These examples illustrate how the Lagrangian framework can be adapted to diverse scenarios, reinforcing the versatility of the formalism.

Common Student Challenges in Chapter 3

Despite its elegance, the Lagrangian approach can present obstacles for learners. Some of the most frequent difficulties include:

1. **Choosing Appropriate Coordinates:** Selecting a convenient set of generalized coordinates is crucial, yet students often default to Cartesian coordinates, missing simplifications.
2. **Deriving the Lagrangian:** Accurately expressing kinetic and potential energies in terms of generalized coordinates requires careful algebra.
3. **Identifying Cyclic Variables:** Recognizing which coordinates are absent from the Lagrangian can be non-obvious, especially in coupled systems.
4. **Solving Coupled Differential Equations:** The resulting Euler–Lagrange equations can be intertwined, demanding techniques such as matrix diagonalization or normal mode analysis.
5. **Applying Constraints:** Systems with holonomic constraints often require Lagrange multipliers, a topic that can feel abstract until practiced.

Addressing these challenges is essential for mastering Chapter 3 and for progressing to the Hamiltonian formalism in subsequent chapters.

Goldstein Solutions for Chapter 3 Problems

Problem 1: Simple Harmonic Oscillator

Problem Statement: A mass m attached to a spring with force constant k oscillates horizontally. Derive the

equation of motion using the Lagrangian method.

Solution:

1. Choose Generalized Coordinate: The natural choice is the displacement x from equilibrium.
2. Express Kinetic and Potential Energies:

$$\text{KE: } T = \frac{1}{2} m \dot{x}^2$$

$$\text{3. PE: } V = \frac{1}{2} k x^2$$

Compute derivatives: $\frac{\partial L}{\partial \dot{x}} = m \dot{x}$ and $\frac{\partial L}{\partial x} = -kx$. Then, $\frac{d}{dt}(m \dot{x}) + kx = 0$.

Problem 2: Particle in a Central Potential

Problem Statement: A particle of mass m moves under a central potential $V(r) = -\frac{\alpha}{r}$. Use polar coordinates to derive the effective one dimensional equation for the radial motion.

Solution:

1. Generalized Coordinates: r and ϕ . The kinetic energy in polar coordinates is $T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\phi}^2)$.
2. Lagrangian: $L = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\phi}^2) + \frac{\alpha}{r}$.
3. Identify Cyclic Coordinate: ϕ . Explicitly, so its conjugate momentum $p_\phi = \frac{\partial L}{\partial \dot{\phi}} = m r^2 \dot{\phi}$ is conserved (angular momentum L_z).
4. Express $\dot{\phi}$ in Terms of r : $\dot{\phi} = L_z / (m r^2)$.
5. Effective Lagrangian for r : Substitute $\dot{\phi}$ back: $L_{\text{eff}} = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} L_z^2 / (m r^2) + \frac{\alpha}{r}$.
6. Euler-Lagrange for r : $\frac{d}{dt} \left(\frac{\partial L_{\text{eff}}}{\partial \dot{r}} \right) = \frac{\partial L_{\text{eff}}}{\partial r}$ gives $m \ddot{r} = -\frac{L_z^2}{m r^3} - \frac{\alpha}{r^2}$.
7. Effective Potential: The radial equation can be written as $m \ddot{r} = -\frac{dV_{\text{eff}}}{dr}$, $V_{\text{eff}}(r) = \frac{L_z^2}{2 m r^2} - \frac{\alpha}{r}$.

Problem 3: Rigid Body Rotation

Problem Statement: A uniform disk of mass M and radius R rotates about its symmetry axis with angular velocity ω . Derive the Lagrangian and equation of motion for ω .

Solution:

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