

classical mechanics iii 8 09 fall 2014 assignment 1

Published: September 27, 2026 | Index ID: #-

Classical Mechanics III 8 09 Fall 2014 Assignment 1 – A Deep Dive

Classical mechanics is the cornerstone of physics, providing the mathematical framework that describes the motion of everyday objects and the dynamics of celestial bodies. For students enrolled in the Fall 2014 semester of the “Classical Mechanics III” course (Course Code 8 09), Assignment 1 presents a series of challenging problems designed to reinforce core concepts such as Lagrangian mechanics, Hamiltonian formalism, and advanced applications of Newtonian dynamics. This article offers a comprehensive guide to understanding, tackling, and mastering the assignment, blending conceptual explanations, step by step solutions, and study strategies that will help you excel.

Course Context and Learning Objectives

What the Course Covers

Classical Mechanics III builds on the foundations laid in earlier mechanics courses. Topics include:

- Variational principles and the Euler–Lagrange equation
- Generalized coordinates and constraints
- Conservation laws derived from symmetries
- Hamiltonian mechanics and canonical transformations
- Small oscillations and normal modes
- Applications to rigid bodies, central forces, and field theory

Students are expected to develop both analytical problem solving skills and a deep conceptual understanding of the underlying physics.

Assignment 1 Learning Goals

Fall 2014 Assignment 1 focuses on the following objectives:

1. Applying the Euler–Lagrange formalism to non trivial systems.
2. Deriving Hamiltonians from given Lagrangians.
3. Identifying conserved quantities through Noether's theorem.
4. Analyzing small oscillations and normal mode decomposition.
5. Interpreting physical implications of mathematical results.

Mastering these goals prepares students for advanced topics like quantum mechanics, statistical mechanics, and general relativity.

Detailed Breakdown of Assignment 1 Problems

Problem 1: Lagrangian for a Pendulum with a Moving Pivot

Statement: A simple pendulum of length L and mass m is attached to a pivot that moves horizontally with a prescribed function $x(t) = A \sin(\omega t)$. Derive the Lagrangian, find the equation of motion, and discuss the effect of the pivot motion on the pendulum's dynamics.

Key Concepts: Non inertial reference frames, generalized coordinates, time dependent constraints.

Solution outline:

1. Choose generalized coordinate θ , $\theta \in \mathbb{R}$ (vertical).
2. Express the position vector of the mass in the inertial frame:

Pivot position: $\mathbf{r}_p = (x(t), 0)$

3. Mass position: $\mathbf{r}_m = (x(t) + L \sin \theta, L \cos \theta)$

- Differentiate $\mathbf{r}_m$ with respect to time, accounting for $dx/dt = A \cos(2\pi f t)$.
- Result: $v^2 = [A \cos(2\pi f t) + L \dot{\theta} \sin \theta]^2 + [L \dot{\theta} \cos \theta]^2$
- $T = \frac{1}{2} m v^2$
- Gravitational potential: $V = m g L (1 - \cos \theta)$
- " L " θ BÖGB, $\hat{A} \rightarrow \theta$ $\rightarrow C$, $= 0$
- After algebra, obtain differential equation:
 $m L^2 \ddot{\theta} + 2 \dot{\theta} \dot{A} L \cos \theta + m L A \cos(2\pi f t) \cos \theta = 0$
- The term $m L A \cos(2\pi f t) \cos \theta$, &W &W6VçG2 ametric driving force due to the pivot's motion.
- For small angles, linearize: $\theta \ll \pi/2$, $\hat{A} v \rightarrow f \rightarrow \theta$
 $\ddot{\theta} + (A \cos(2\pi f t) \hat{A} \cos \theta) = 0$, a Mathieu type equation.

Problem 2: Hamiltonian for a Particle in a Central Potential

Statement: Derive the Hamiltonian for a particle of mass m moving in a spherically symmetric potential $V(r)$, and identify the conserved angular momentum components.

Key Concepts: Canonical coordinates, Legendre transform, symmetries.

Solution outline:

1. Use spherical coordinates (r, θ, ϕ) .
2. Compute kinetic energy:
 $T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \sin^2 \theta \dot{\phi}^2)$
3. Write Lagrangian:
 $L = T - V(r)$.
4. Find conjugate momenta:

$p_r = \frac{\partial L}{\partial \dot{r}} = m \dot{r}$

5. $p_\theta = \frac{\partial L}{\partial \dot{\theta}} = m r^2 \dot{\theta}$

6. $p_\phi = \frac{\partial L}{\partial \dot{\phi}} = m r^2 \sin^2 \theta \dot{\phi}$

• $H = p_r \dot{r} + p_\theta \dot{\theta} + p_\phi \dot{\phi} - L$

• Substitute velocities from momenta:

$\dot{r} = p_r / m$, $\dot{\theta} = p_\theta / (m r^2)$, $\dot{\phi} = p_\phi / (m r^2 \sin^2 \theta)$

• Result:

$H = (p_r^2)/(2m) + (p_\theta^2)/(2m r^2) + (p_\phi^2)/(2m r^2 \sin^2 \theta) + V(r)$.

• Because V depends only on r , the Hamiltonian is invariant under rotations.

• Thus p_ϕ & $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ are conserved.

Problem 3: Small Oscillations of a Coupled Oscillator System

Statement: Two masses m_1 and m_2 are connected by three springs with constants k_1, k_2, k_3 , forming a linear chain. Derive the normal modes and frequencies for small displacements from equilibrium.

Key Concepts: Mass matrix, stiffness matrix, eigenvalue problem.

Solution outline:

1. Define generalized coordinates x_1, x_2 as displacements of each mass.

2. Write kinetic energy:

$$T = \frac{1}{2} (m_1 \dot{x}_1^2 + m_2 \dot{x}_2^2).$$

3. Write potential energy:

$$V = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 (x_2 - x_1)^2 + \frac{1}{2} k_3 x_2^2.$$

4. Form the mass matrix $M = \text{diag}(m_1, m_2)$ and stiffness matrix $K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix}$.

5. Set up eigenvalue problem:

$$K u = \omega^2 M u$$

6. Compute determinant:

$$\det(K - \omega^2 M) = 0$$

7. Solve for ω^2 :

$$\omega^2 = \frac{(k_1 + k_2 + k_3)(m_1 + m_2) \pm \sqrt{[(k_1 + k_2 + k_3)(m_1 + m_2)]^2 - 4(m_1 m_2)(k_1 k_3)}}{2 m_1 m_2}.$$

8. Interpret normal modes:

In phase mode: masses move together, dominated by the outer springs.

Out of phase mode: masses move oppositely, dominated by the middle spring.

Problem 4: Noether's Theorem in a Rotating Frame

Statement: Show that the conservation of angular momentum arises from rotational symmetry in a Lagrangian describing a particle in a rotating reference frame with angular velocity ω .

Key Concepts: Symmetry transformations, conserved currents.

Solution outline:

1. Write the Lagrangian in the rotating frame:

$$L = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\phi}^2) - V(r)$$

2. Consider an infinitesimal rotation about the z axis: $\phi \rightarrow \phi + \delta\phi$

3. Compute the variation of coordinates:

$$\delta \mathbf{r} = \hat{z} \times \mathbf{r} \delta\phi$$

4. Calculate the change in Lagrangian:

$$\delta L = m r \dot{\phi} \delta\phi$$

5. Using Euler-Lagrange, show that the total derivative of the angular momentum component $L_z = m r^2 \dot{\phi}$ vanishes.

6. Conclude that L_z is conserved, reflecting rotational symmetry.

Common Challenges and How to Overcome Them

- Misidentifying Generalized Coordinates: Always check if the chosen coordinate respects the constraints of the system. For example, using θ for a pendulum ensures the length constraint is automatically satisfied.

Baca Juga (Artikel Terkait)

- [chuunibyou english](http://basicgravity.com/chuunibyou-english.pdf)

URL: <http://basicgravity.com/chuunibyou-english.pdf>

- [churchill we shall fight on the beaches](http://basicgravity.com/churchill-we-shall-fight-on-the-beaches.pdf)

URL: <http://basicgravity.com/churchill-we-shall-fight-on-the-beaches.pdf>

- [churchill roy jenkins](http://basicgravity.com/churchill-roy-jenkins.pdf)

URL: <http://basicgravity.com/churchill-roy-jenkins.pdf>

- [churchill roosevelt stalin the war they](http://basicgravity.com/churchill-roosevelt-stalin-the-war-they.pdf)

URL: <http://basicgravity.com/churchill-roosevelt-stalin-the-war-they.pdf>

Tags: #classical #mechanics #fall #2014 #assignment

