

conditional probability examples and solutions

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Conditional Probability Examples and Solutions: A Comprehensive Guide

Conditional probability is a cornerstone of statistics, data science, and everyday decision making. Whether you're a student tackling a math exam, a data analyst interpreting survey results, or a curious reader trying to understand why a certain medical test is reliable, the concept of "probability given a condition" is everywhere. In this article, we'll walk through the fundamentals of conditional probability, explore a variety of real world examples, and solve each one step by step. By the end, you'll not only grasp the theory but also be able to apply it confidently in any situation that involves uncertainty.

What Is Conditional Probability?

Definition

Conditional probability, denoted as $P(A | B)$, is the probability of event **A** occurring under the assumption that event **B** has already occurred. It refines our understanding of risk by incorporating additional information.

The Formula

The basic formula for conditional probability is:

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

where $P(A \cap B)$ is the probability that both events happen together, and $P(B)$ is the probability that the condition is met. The denominator must be greater than zero; otherwise, the conditional probability is undefined.

Why It Matters

In everyday life, we rarely face events in isolation. We always have background information that shapes our expectations. Conditional probability formalizes how that background information changes our predictions.

Common Misconceptions and Clarifications

- "Conditional probability is the same as joint probability." – Not true. Joint probability is $P(A \cap B)$, while conditional probability divides the joint probability by the probability of the condition.
- "If two events are independent, their conditional probability equals their marginal probability." – Correct. For independent events, $P(A | B) = P(A)$.
- "A high probability of B always means a high probability of A." – No. The condition may be unrelated to A.

Real-World Examples and Their Solutions

1. Medical Testing: Understanding False Positives

Suppose a disease affects 1% of a population. A diagnostic test has 95% sensitivity (true positive rate) and 90% specificity (true negative rate). What is the probability that a person actually has the disease given a positive test result?

Solution

1. Define events: D = person has disease, T+ = test is positive.

2. Given: $P(D) = 0.01$, $P(T+ | D) = 0.95$, $P(T+ | \neg D) = 0.10$ (since specificity is 90%, false positive rate is 10%).

3. Compute $P(T+)$ using the law of total probability:

$$P(T+) = P(T+ | D) \times P(D) + P(T+ | \neg D) \times P(\neg D)$$

$$= 0.95 \times 0.01 + 0.10 \times 0.99$$

$$= 0.0095 + 0.099$$

$$= 0.1085.$$

4. Apply Bayes' theorem:

$$P(D | T+) = P(T+ | D) \times P(D) / P(T+)$$

$$= 0.95 \times 0.01 / 0.1085$$

$$= 0.0875 \text{ (8.75\%).}$$

Even with a highly accurate test, the probability that a positive result indicates disease is under 9% because the disease is rare.

2. Card Game: Probability of Drawing a Queen After a Red Card

In a standard 52 card deck, what is the probability of drawing a queen after you have already drawn a red card (hearts or diamonds) and not replaced it?

Solution

1. Let R be the event "first card is red." Q is "second card is a queen."

2. We need $P(Q | R) = P(Q \cap R) / P(R)$.

3. Compute $P(R) = 26/52 = 0.5$.

4. To find $P(Q \cap R)$, consider two scenarios:

First card is a red queen (2 possibilities). Then the second card can be any of the remaining 51 cards, but we only care if it is a queen. Since the first queen is removed, there are 3 queens left among 51 cards. Probability = $(2/52) \times (3/51)$.

5. First card is a red non queen (24 possibilities). Then the second card can be any queen among 51 cards. Probability = $(24/52) \times (4/51)$.

So, after seeing a red card, the chance of drawing a queen next is about 7.7%.

3. Weather Forecasting: Rain Probability After a Sunny Day

A weather model indicates that the probability of rain on any given day is 30%. However, if the previous day was sunny, the probability of rain drops to 15%. If you observe a sunny day followed by a rainy day, what is the conditional probability that the second day will be rainy given that the first was sunny?

Solution

Here, the condition is already specified by the model: $P(\text{Rain} | \text{Sunny}) = 0.15$, the conditional probability is 15% directly. The model already incorporates the dependency between consecutive days.

4. Manufacturing Defect Detection

A factory produces 10,000 widgets per day, with a defect rate of 2%. Quality control samples 100 widgets randomly. What is the probability that at least one defective widget is found in the sample?

Solution

1. Let D = "a widget is defective." S = "sample contains at least one defective." We want $P(S)$.

2. It is easier to compute the complement: $P(\neg S)$ = probability that none of the 100 sampled widgets are defective.

3. Since sampling is without replacement, the probability that the first widget is non defective is $(10,000 - 1) / 10,000 = 0.99$. After removing one non defective, the probability for the second is $(9,999 - 1) / 9,999 = 0.9999$. Continue for 100 draws.

4. Product of these 100 probabilities yields $P(\neg S) = (0.99)^{100} \approx 0.3697$.

5. Therefore, $P(S) = 1 - 0.3697 \approx 0.6303$ (63.03%).

The conditional probability that the sample contains at least one defect is high because even a small defect rate accumulates over 100 items.

5. Social Media Algorithm: Likelihood of Content Going Viral

A platform's algorithm estimates that the probability of a post being shared by a user is 5%. If a user has 200 followers, what is the probability that at least one follower shares the post? Assume independence.

Solution

- Let S = "a specific follower shares." V = "at least one follower shares." We need $P(V)$.
- Complement: probability that no follower shares is $(1 - 0.05)^{200} \approx 0.3585$.
- Compute: $0.95^{200} \approx e^{(200 \times \ln 0.95)} \approx e^{(-10.26)} \approx 0.000035$.
- Thus, $P(V) = 1 - 0.000035 \approx 0.999965$ (99.9965%).

Even with a low individual sharing probability, the sheer number of followers makes viral sharing almost guaranteed.

6. Spam Email Detection

Suppose 2% of all emails are spam. A spam filter flags 90% of spam emails (true positive) and mistakenly flags 5% of legitimate emails (false positive). What is the probability that an email flagged by the filter is actually spam?

Solution

- Define events: S = "email is spam," F = "email flagged."
- Given: $P(S) = 0.02$, $P(F | S) = 0.90$, $P(F | \neg S) = 0.05$.
- Compute $P(F) = 0.90 \times 0.02 + 0.05 \times 0.98 = 0.018 + 0.049 = 0.067$.

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